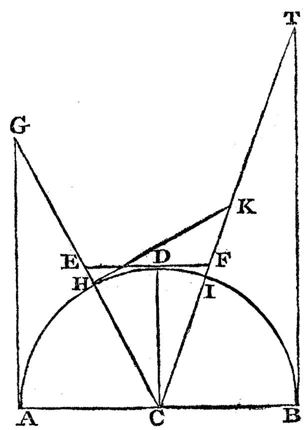
VIII. *On a new Property of the Tangents of the three Angles of a Plane Triangle. By Mr.* William Garrard, *Quarter Master of Instruction at the Royal Naval Asylum at Greenwich. Communicated by the Astronomer Royal.*

Read February 11, 1808.

PROPOSITION I. In every acute angled plane triangle, the sum of the three tangents of the three angles multiplied by the square of the radius, is equal to the continued product of the tangents.

Demonstration.—Let AH, HI, and IB be the arches to represent the given angles; and AG, HK, and BT be their tangents, put r the radius, AG $= a$, and BT $= b$,



Then $\frac{r^2}{a}$ and $\frac{r^2}{b}$ will be the tangents of HD and DI.

Now by Prop. VIII. Sect. I. Book I. EMERSON's Trigonometry,

As radius square—product of two tangents
Is to radius square,
So is the sum of the tangents
To the tangent of their sum.

$$\therefore r^2 - \frac{r^4}{ab} : r^2 :: \frac{r^2}{a} + \frac{r^2}{b} : \frac{r^2a + r^2b}{ab - r^2} = \text{HK};$$

therefore $a + b + \frac{r^2a + r^2b}{ab - r^2} = \frac{a^2b + ab^2}{ab - r^2} =$ the sum of the three tangents,

and $\frac{a^2b + ab^2}{ab - r^2} \times r^2 = ab \times \frac{r^2a + r^2b}{ab - r^2} =$ their continued product.

Q. E. D.

PROPOSITION II. In every obtuse angled plane triangle, the sum of the three tangents of the three angles multiplied by the square of radius, is equal to their continued product.

Demonstration.—Let AH be an obtuse arc, and HE, ED the other two.

Then BF, ED, and AG are the three tangents.

Put BF $= t$ and DE $= u$ radius $= r$, then per trigonometry, as before,

$r^2 \times \frac{t+u}{r^2 - tu} = $ BT;

But $-$ BT $=$ AG $= - \frac{t+u}{r^2 - tu} \times r^2$.

Wherefore $t + u - \frac{t+u}{r^2 - tu} \times r^2 =$ the sum of the three tangents, which being reduced

is $= - tu \times \frac{t+u}{r^2 - tu}$, and multiplied into r^2 is equal to

$tu \times - \frac{t+u}{r^2 - tu} \times r^2 =$ the product.

Q. E. D.